16.4 Videos Guide

16.4a

Theorem (statement and partial proof):

- Green's Theorem (essentially the Fundamental Theorem of Calculus for double integrals): For a region D in $\mathbb{R}^2$ bounded by a positively oriented, simple, closed curve C and a vector field $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$, if P and Q have continuous partial derivatives on an open region containing D, then
$\int_C \mathbf{F} \cdot d\mathbf{r} = \oint_C P\,dx + Q\,dy = \int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

16.4b

- Notation: $C = \partial D$ is the simple, positively oriented boundary curve of D. The symbol $\oint_C$ is used to indicate positive orientation.

Exercises:

- Evaluate the line integral by two methods: (a) directly and (b) using Green's Theorem.
    - $\oint_C y\,dx - x\,dy$, C is the circle with center the origin and radius 4

16.4c

- $\oint_C x^2y^2\,dx + xy\,dy$, C consists of the arc of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ and the line segments from $(1, 1)$ to $(0, 1)$ and from $(0, 1)$ to $(0, 0)$

16.4d

- Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Check the orientation of the curve before applying the theorem.)
$\mathbf{F}(x, y) = \langle e^{-x} + y^2, e^{-y} + x^2 \rangle$, C consists of the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$ and the line segment from $(\pi/2, 0)$ to $(-\pi/2, 0)$

- Green's Theorem and regions with holes